White Paper on FITACF

Update on FITACF for Sept. 2003

By

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Introduction

Two issues have come up over the past few years concerning the results obtained from FITACF. The first is the identification of ground-scatter and the second is the way in which we estimate the error in the velocity.

I presented a concept for improving the velocity error estimation in the tutorial on FITACF that I presented at the Valdez SuperDARN meeting. In the summer following the Valdez meeting (Aug. 2002), I coded up the new approach, but I was unable to test and debug the code. Now, at the end of the summer of 2003 I have had the time to test the code and verify the results.

During the summer of 2003, Prof. Gerard Blanchard, from Southeastern Louisiana University, visited APL with an undergraduate student. Under the direction of Dr. Blanchard and myself, the student, Mr. Stephen Sundeen, analyzed a large block of radar data containing both ground scatter, ionospheric scatter and cases of mixed scatter. Mr. Sundeen was able to complete a preliminary analysis of the characteristics of ground scatter vs ionospheric scatter, and I have implemented this analysis in the new version of FITACF. Mr. Sundeen intends to continue working on the analysis of the ground scatter vs ionospheric scatter characteristics over the coming academic year and we expect to have a definitive analysis ready in time for the next SuperDARN meeting.

In the following section of this white paper I will present a brief review of the results from Mr. Sundeen's ground scatter analysis and describe how it differs from the older methods we used for flagging ground scatter. This section will be followed by a description of the calculation of the velocity error. The penultimate section will provide a comparison of the results obtained from the current version of FITACF and the new version. The final section will provide a conclusion and recommendation.

Ground Scatter

The original definition of ground scatter was based on work done by J.-P. Villain and myself when JPV was spending a year at APL (1984?). The definition that we came up with involved the velocity, the spectral width, and the backscattered power. We found that at lower powers there was a tendency for the spectral width to increase and if we didn't take that fact into account we would identify an excess number of weak ground scatter echoes as being ionospheric scatter.

At some point there was a change made to this original code. No one seems to know who initiated the change or exactly why it was done. In any case, in the current version of fitacf there is no attempt to take the power into account and instead the algorithm makes use of the velocity and spectral width as well as the error estimates in those two parameters. The algorithm is very simple:

1. Calculate $v - \Delta v$. If the result is less than 0 set it equal to 0.
2. Calculate $w - \Delta w$. If the result is less than 0 set it equal to 0.
3. If the value from step 1 is less than 30 m/s and the value from step 2 is less than 35 m/s then set the ground scatter flag.

This algorithm is problematical for several reasons. First, of course, it doesn't seem to be based on any sort of careful analysis. Second, as will be discussed below, the values we have been using for Δv are the result of what I consider to be a flawed method of determining the error. Third, a large value in the errors could lead to a point being flagged as ground scatter even if it had a large velocity and spectral width.

Sundeen-Blanchard-Baker Analysis

We chose data from the Kapuskasing and Saskatoon radars for a total of 12 days. Three days were chosen from near the spring equinox, three from the summer solstice, three from the fall equinox and three from the winter solstice. In each of the 12 days chosen there was a large amount of scatter and we had both ground scatter and ionospheric scatter. A set of 2-D histograms was calculated giving the number of observations as a function of abs(velocity) and spectral width (we used the width_l, exponential decay model). There was one histogram for each season. We prepared histograms for three different ranges of backscattered power, $P < 3$ dB, 3 dB $< P < 6$ dB, 6 db $< P$. In all cases the histograms were nearly identical in shape. We found (as we expected) there were two populations, one with low velocity and low spectral width and one with a full range of velocities and spectral widths. The cross-over point between the two populations where the probabilities were equal was estimated and this cross-over point can be used as the boundary of the two populations.

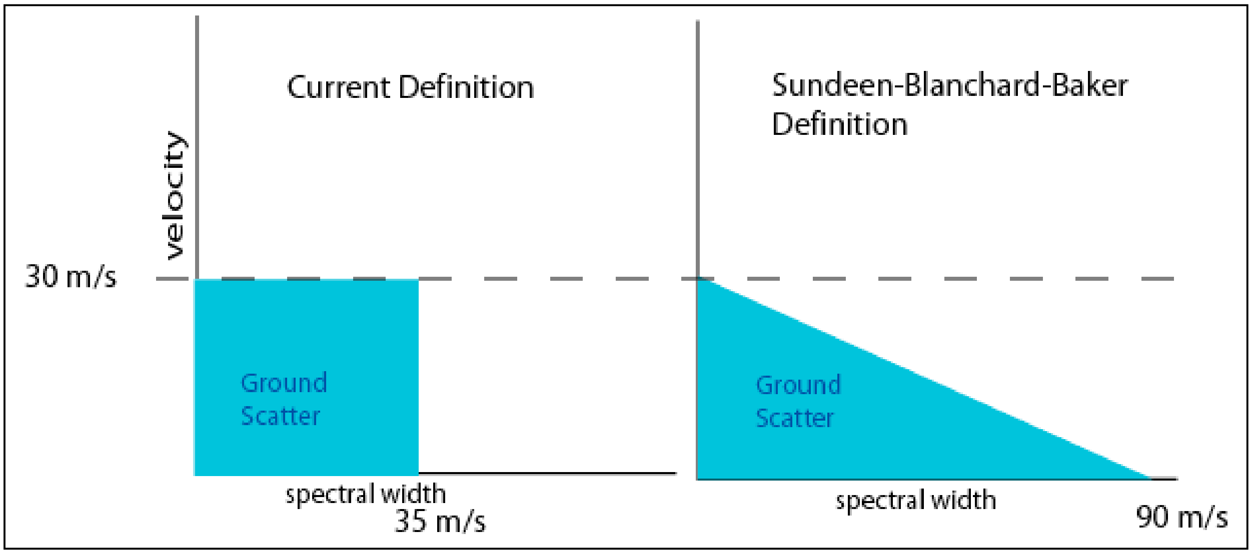

Figure 1. A schematic of the current ground scatter definition and the new definition.

The new analysis indicates that there is a relationship between spectral width and velocity that needs to be recognized when attempting to distinguish between ground scatter and ionospheric scatter. The new definition allows some points with relatively high spectral widths to be classified as ground scatter provided the velocity is sufficiently low. On the other hand, there is a significant region where the new definition would indicate was ionospheric scatter that would be considered ground scatter under the current definition. It must also be remembered that the current definition is subtracting the velocity error from the velocity and the spectral width error from the spectral width before making the classification.

When I implemented the new definition and compared the results with the current definition it was clear that a significant number of points that were imbedded in regions of ground scatter were now being identified as ionospheric scatter. A careful examination of these points indicated that in all cases these apparently misidentified points had large errors in either the velocity or spectral width. A naïve approach to fixing the problem would be to take the same route used by the current definition and simply subtract the errors, replacing the subtracted result with 0 if the subtraction would drive the value negative. However, we can take a somewhat more sophisticated approach. Let us define the ground scatter function *g*.

$$g(v,w) = v - v_{\max} + \frac{v_{\max}}{w_{\max}} w \qquad (1)$$

If the value of g is ≤ 0 then we consider the point to be a member of the ground scatter population, while if $g > 0$ it will be considered ionospheric. Now, what is the estimated error in g? Since the velocity is derived from fitting the ACF phases while the spectral width is derived from fitting the power as a function of lag, we may consider the two errors to be independent.[1] In that case we can use the standard approach for the calculation of the propagation of error and we will have

$$\left(\frac{\Delta g}{g}\right)^2 = \left(\frac{\Delta v}{v}\right)^2 + \left(\frac{\Delta w}{w}\right)^2 \qquad (2)$$

We can now test the value of g - Δg instead of g by itself. Although this approach to including the errors in the process of determining the ground-scatter flag is probably more appropriate than the current method, it is still biased toward flagging data as ground scatter. In the data comparison section below we will examine the results from the different approaches in more detail. It should be noted, however, that FITACF sets the ground-scatter flag as a convenience to the user. The value of the parameters and their errors is not changed by the flag setting. Thus, the user is always able to ignore the ground-scatter flag and attempt to identify what is and what is not ground scatter by any method he/she pleases.

Velocity Error

If you have *a priori* knowledge of the errors in your measurements, the estimation of the error in a quantity derived from those measurements is relatively straightforward and many books deal extensively with this issue. When the measurement errors are not known *a priori* it may possible to estimate the errors based on the results of the data analysis.

Error in Measuring Phase

At past SuperDARN meetings I have given presentations demonstrating that under reasonable assumptions about the nature of the noise, the error in a measurement of the phase at a given lag is inversely proportional to the power at that lag. This is certainly what we might expect. As the power increases above the noise level our ability to measure the phase of the incoming signal obviously should improve.

Error in Measure Power

It is also intuitive that the error in actually determining the power at any given lag should also go down as the signal intensity increases. Again, at previous SuperDARN meetings I have shown that when we take the log of the power, which is what we need for doing the power fits, the error in the log is proportional to the power at that lag. Note that in both the phase case and the log(power) case the error is proportional to the power itself, not the power in dB, which is a logarithmic quantity.

Characterization of the Measurement Errors

The error analysis of both the phase fit and the power fit begins with the following assumption: the error in the measurement (expressed in terms of the standard deviation) can be expressed as follows:

[1] Actually, we should go through all the data used to determine the new definition of ground scatter and calculate the covariance matrix for the velocity and width. Hopefully we will be able to do that analysis in time for the 2004 SuperDARN meeting.

$$\sigma_i = \sigma \frac{\langle P \rangle}{P_i} \tag{3}$$

where P_i is the linear power the lag, and <*P*> is the average power over all the good lags in the ACF. Turning now to the process of fitting the phase of the ACF to get the velocity, and following *Bevington and Robinson*[2] we note that we want to minimize χ^2 with respect to the angular frequency ω, where

$$\chi^2 = \sum \left[\frac{\varphi_i - \omega t_i}{\sigma_i} \right]^2 \tag{4}$$

and φ_i is the measured phase at lag-i and t_i is the lag-time. The sum is to be done over all the "good" lags. One special point needs to be made here. The weighting of the points in equation (4) is $(1/\sigma_i)^2$ and thus the weighting in terms of the power comes in as the power squared. This weighting is true not only for the phase fit but also for the power fit. In the current version of FITACF there was an error and the weighting was only being done by the power and not the power squared. As will be seen in the section on data comparisons this bug has only a minor effect on the primary parameters, but has a more significant effect on the error estimates.

<u>*True Phase vs Measured Phase*</u>

There is one major complication to analyzing the phase data. The measured phase is always in the range $-\pi$ to $+\pi$. The true phase of the ACF is presumed to be a linear function of time. We therefore need to determine how to add multiples of 2π to the measured phases.

We must presume that the velocity is below that aliasing velocity and that the phase difference between one lag and the next in the ACF is between $\pm\pi$. We should therefore be able to get a good estimate of the frequency by adding together all the phase differences from all the good lag pairs with a lag difference of 1. In addition to determining an average 1-lag phase difference, we will also be able to determine an error estimate on that value from the standard deviation of the measured 1-lag differences. We thus have an initial guess, ω_0, as well as a high and low value obtained from $\omega_0 \pm \Delta\omega$. From the initial guess of ω_0 we can then predict what the true phase value should be at any time and then add or subtract a sufficient number of multiples of 2π to the measured phase so that the resulting total phase lies within $\pm\pi$ of the predicted value. Actually the process of doing this is somewhat more complicated, but I won't go into the details here. The end result is that based on our initial guess we determine the "true" phase at each lag and then minimize χ^2 (equation (4)).

We can then repeat the entire process but starting from the high and low initial guesses given by $\omega_0 \pm \Delta\omega$. In most cases the final determination of how to add multiples of 2π turn out to be the same and hence the final determination of ω is the same. We therefore

[2] *Data Reduction and Error Analysis for the Physical Sciences – Second Edition,* Philip R Bevington and D. Keith Robinson, McGraw-Hill, Boston, MA, (1969), pp. 328.

have confidence in the way we have added the multiples of 2π and we can proceed with the analysis of the error in the standard fashion (see below). If, however, we find that one or the other or both of the high/low initial guesses gives us a final result that is different from the result obtained with ω_0 then all we can reasonably say is that the value of ω must lie somewhere between the high and low values of the final fits.

Minimizing χ^2

Let us now return to equation (4) and proceed with the minimization with respect to ω. Setting the derivative of equation (4) with respect to ω equal to zero and using equation (3) for the value of σ_i we have:

$$\omega \sum t_i^2 P_i^2 = \sum \varphi_i t_i P_i^2 \tag{5}$$

We note that the unknown value of σ, along with the value of <P> have disappeared and we can therefore determine the best fit for ω without knowing the actual value of the phase errors. However, in order to estimate the error in our determination of ω we will need to know σ. Luckily, since it doesn't actually come into our determination of ω, we can calculate an estimator for σ directly from the data. We simply calculate the weighted deviation of our data from the fitted values. The result is given in equation (6). The factor n/(n-1) comes from the fact that we have used up one degree of freedom by determining ω.

$$\sigma^2 = \frac{n}{n-1} \frac{\sum P_i^2 (\varphi_i - \omega t_i)^2}{\sum P_i^2} \tag{6}$$

The final estimate for the error in ω is then given by:

$$(\Delta\omega)^2 = \sum \sigma_i^2 \left(\frac{\partial \omega}{\partial \varphi_i} \right)^2 = \sigma^2 \frac{\langle P \rangle^2}{\sum P_i^2 t_i^2} \tag{7}$$

When doing the phase fit for the cross-correlation functions (XCFs) the process is similar but we have to fit an additional parameter, the phase at lag-0. This is just the standard problem of fitting a straight line to the data and the result can be found directly from *Bevington and Robinson.*

Data Comparisons

Overview

Figures 2a-c show range-time-parameter plots for three different versions of FITACF. The data are from beam 8 of the Kapuskasing radar on March 25, 2001. Fig. 2a shows the result from the current version of FITACF, 2b shows the result from the new version of FITACF with no corrections made for the estimated error in the ground-scatter function, *g*, and 2c shows the result from the new version of FITACF but with the estimated error in *g* subtracted from *g*. Note that the data contains a significant number of points that are clearly ionospheric, as well as a large number of points that are clearly ground scatter and finally a significant number of points that may be a mixture and in any case are difficult to accurately categorize.

Comparing 2a and 2b the first thing to be noticed is the large amount of data that was flagged as ground scatter by the current software (2a) that is now considered to be ionospheric scatter. Some regions, such as the poleward patch centered around 10:00 UT show a coherent velocity structure that suggests that this really is a patch of weak, slowly moving ionospheric scatter and the new analysis is correctly determining this. Others, such as the band of "ionospheric" scatter that lies along the poleward edge of the main region of ground scatter, appear to be weak ground scatter signals that are being misinterpreted by the new algorithm.

If we then consider Fig. 2c, the new ground-scatter algorithm but with the error subtracted, we get the initial impression that this is a "cleaner" plot and more consistent with the original plot, Fig. 2a. However, closer inspection reveals that version 2c now flags some regions as ground scatter that seem more likely to be ionospheric scatter. Look particularly at the region that extends from abut 800 km to 1200 km in range during the period from about 02:00 to 10:00 UT. In both 2a and 2b this region shows a well organized pattern, including a small patch of high speed negative velocity at around 07:00 UT.

Turning to a comparison of the magnitudes of the velocity we note that where the data are flagged as ionospheric scatter, the values appear to be very consistent. More detailed comparisons will be made below. Similarly, the values of power and spectral width are all very consistent. We note, of course, that the only difference between 2b and 2c is whether a point is flagged as ground scatter or not.

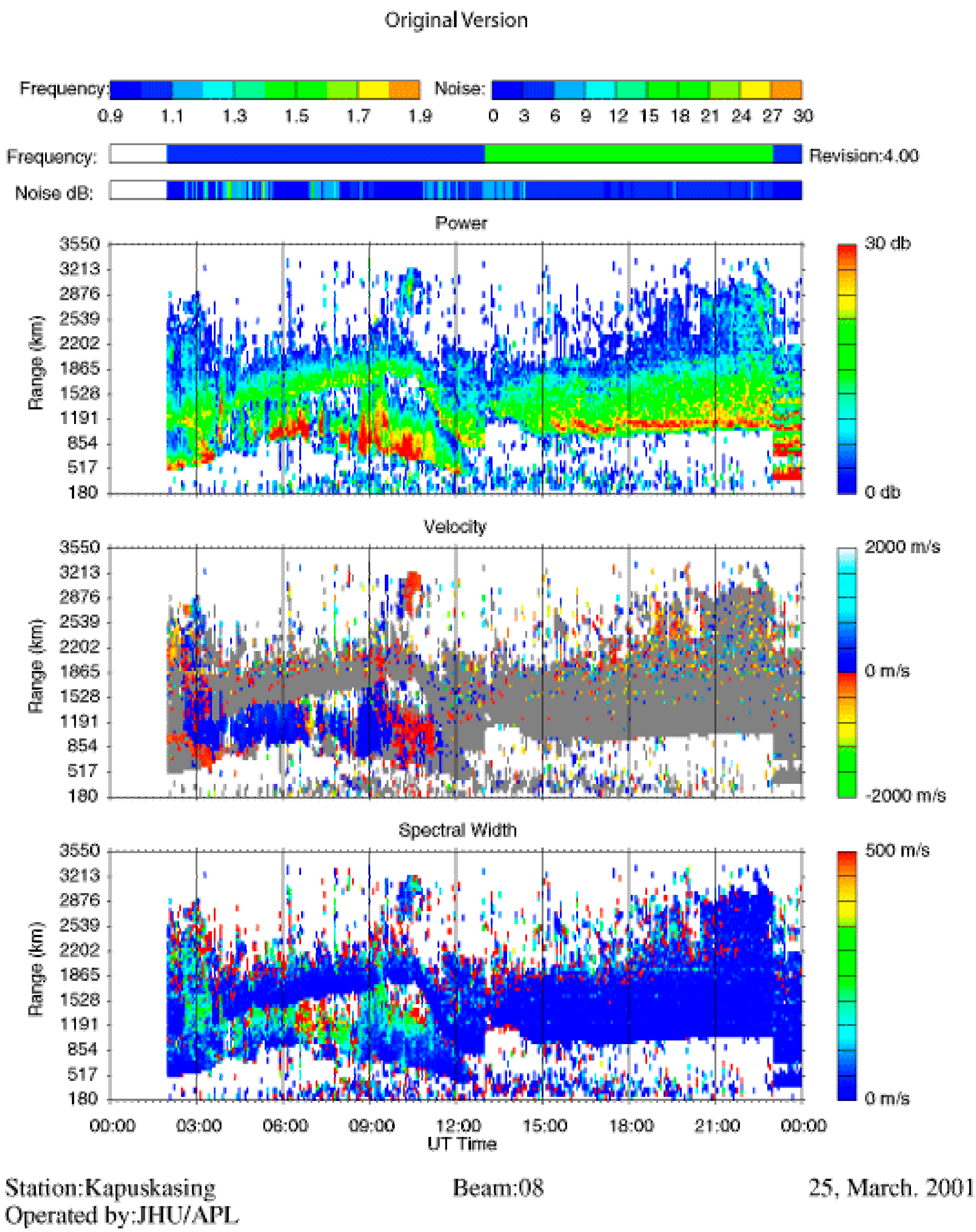

Figure 1a. Results from original version of FITACF

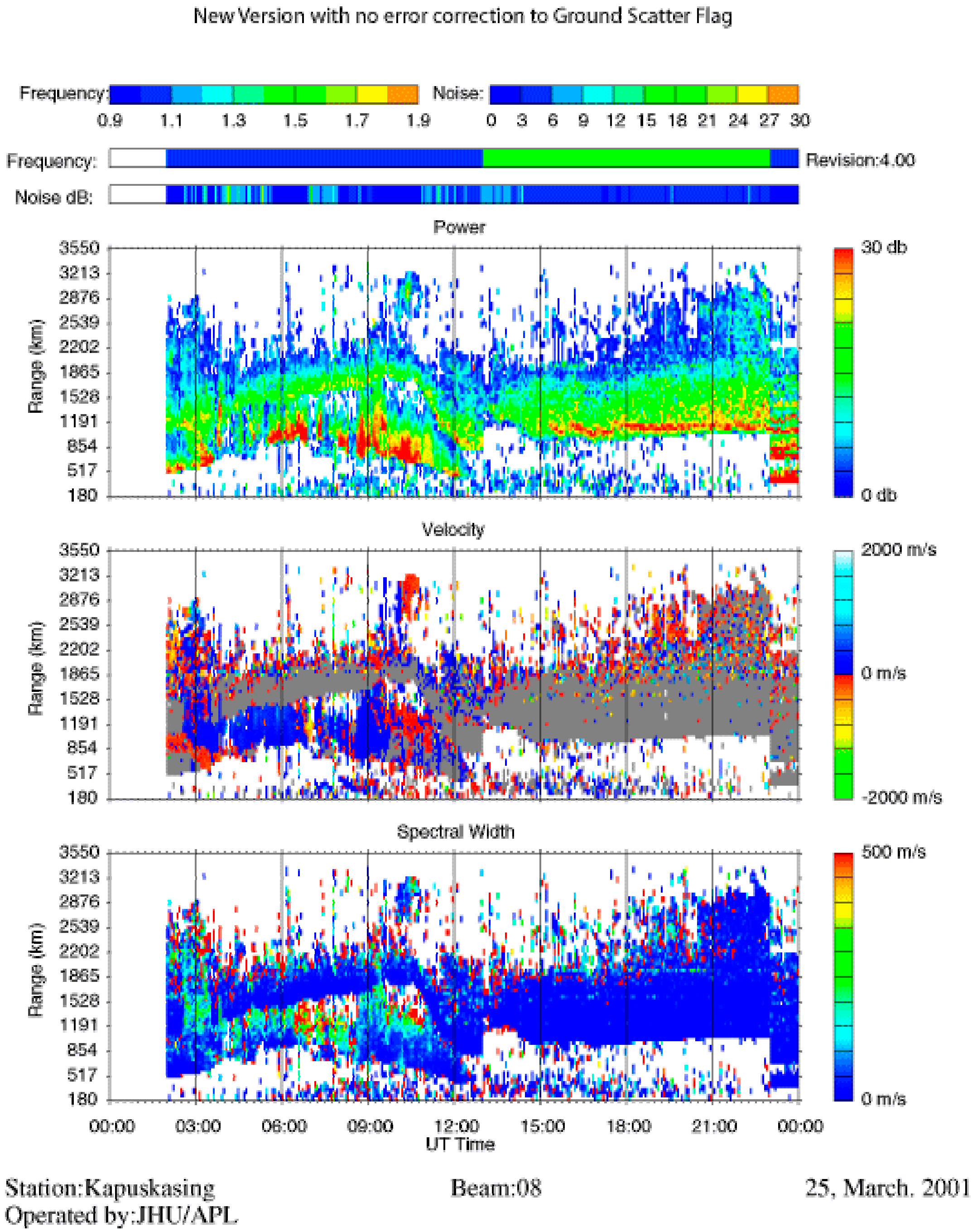

Figure 2b. Results from new version of FITACF with no adjustments made for possible errors in identifying ground scatter.

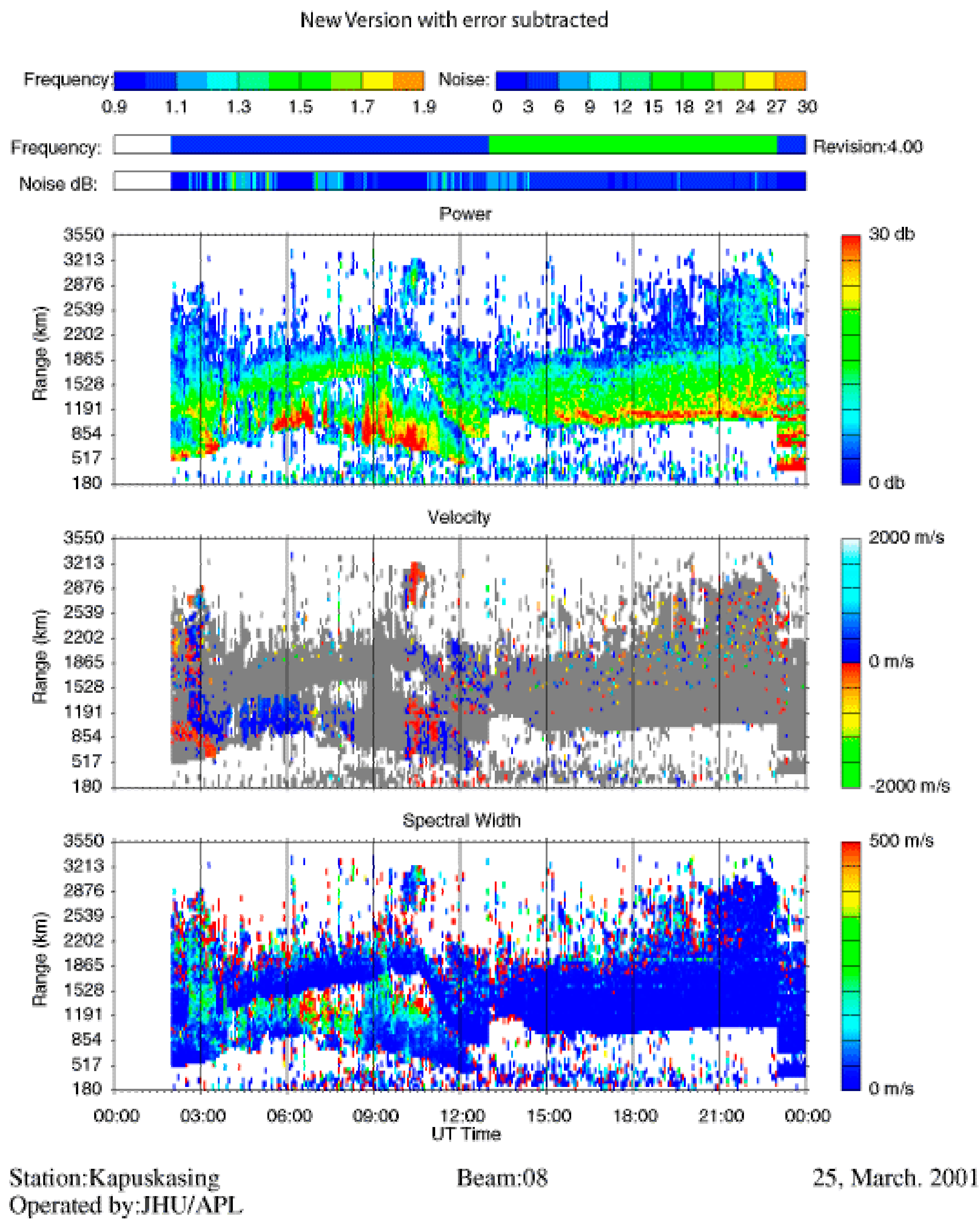

Figure 2c. Results from new version of FITACF with the number of points identified as ground scatter extended to include the all points within the 1σ ground scatter error bar.

Power and Width comparisons

We now turn to the comparison of the two parameters derived from fitting the power-vs-lag profile: the strength of the backscattered power and the spectral width. For the purposes of this comparison we will use the so-called “lambda fit” derived from the assumption that the decrease in power as a function of lag is exponential. Nearly identical results are obtained if one uses the “sigma fit” based on a gaussian decay.

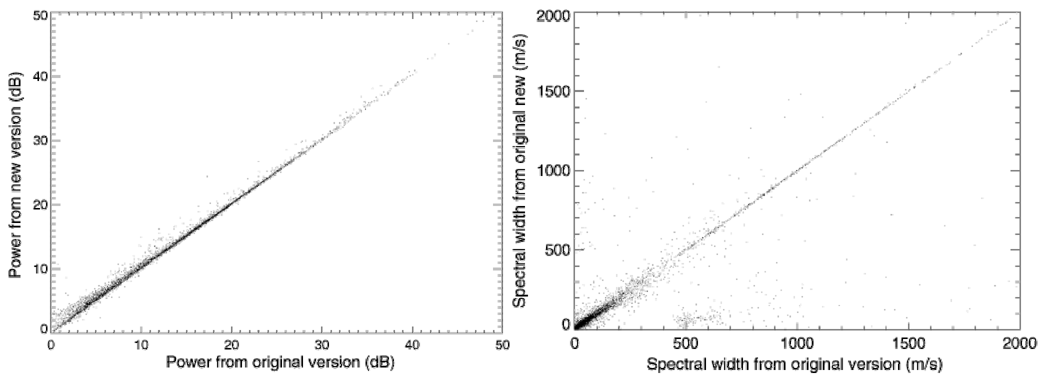

Figure 2. Comparison of backscattered power and spectral widths. The plot shows 10,000 randomly selected points from over 300,000.

As can be seen, the new and old versions give nearly identical results. Only at low powers do we see any significant differences, where the change in the method of weighting the points leads to a slightly larger estimate of the backscattered power.

Velocity comparisons

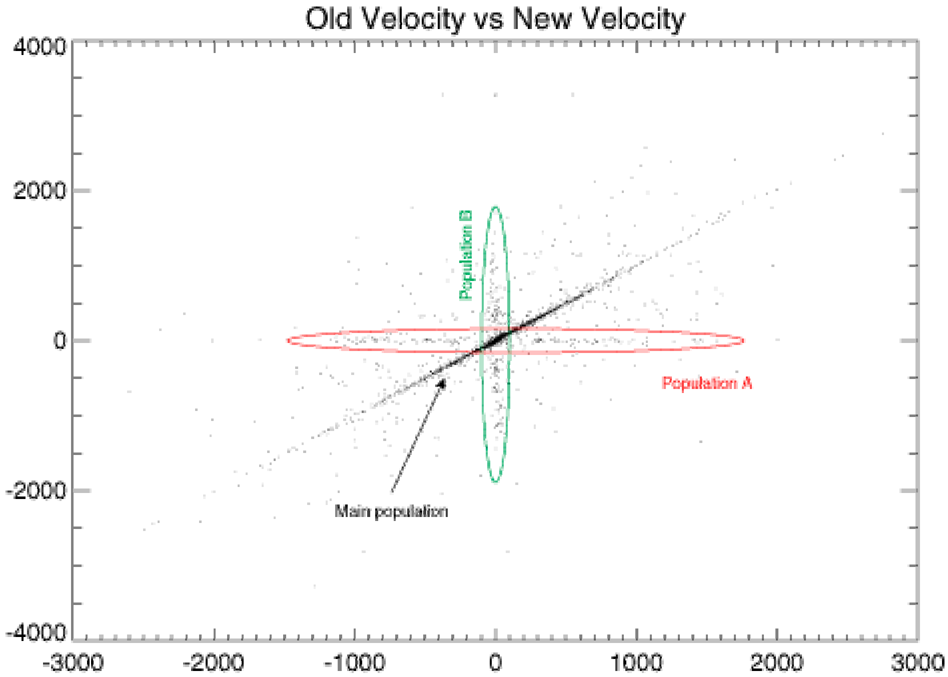

Figure 3. Comparison of velocity determined by original version (horizontal axis) and by the new version (vertical axis)

The vast majority of the points are found within the population referred to here as the "main population." For the points in this population the two estimates of the velocity are in extremely good agreement. There are, however, two distinct populations of points that show large differences. The group referred to in Figure 4 as Population A consists of points that had full range of velocities under the original version of FITACF but have nearly 0 velocity under the new version. Conversely, Population B consists of points that have a full range of velocities under the new version but nearly 0 velocities under the original. These differences come about because of the corrected weighting scheme. In all cases that I have examined in detail, the number of good lags for the points in Pop A and B was small and differences in the weighting resulted in large differences in the initial guess for the frequency. Also, for all the cases examined in detail, the error estimate (see below) under the new version of FITACF was found to be large.

Velocity and velocity error

We now turn to an examination of the velocity and the associated velocity error. It is here, for the first time, that we find major differences between the old and new versions of FITACF. Of course, this is what we expected. The corrected weighting scheme should not cause major differences in the estimates of the primary parameters, but it does make a significant difference in the estimation of the errors. In addition, the old method of estimating velocity error was very *ad hoc* and was not based on sound mathematical analysis.

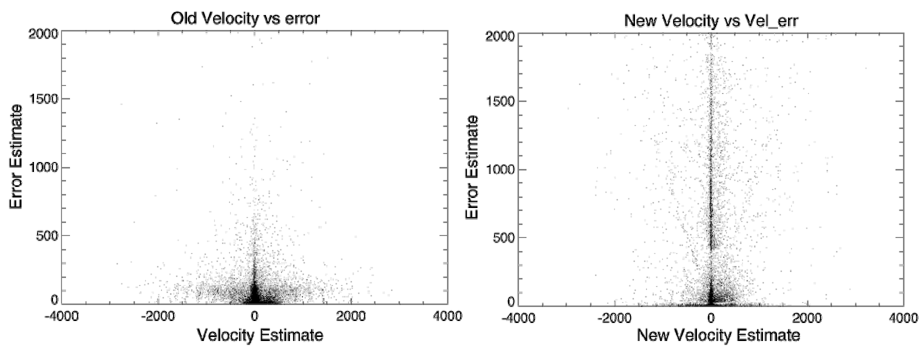

Figure 4. Comparison of velocity vs error for the original FITACF and the new version.

There really isn't much to be said about the old version, since (as noted above) the validity of the error estimate in the old version is questionable. A large majority of the points for the new version have very low errors. Indeed, many points are so close to the x axis that it is difficult to make them out. The interesting feature of the new analysis is the large number of points that lie along the $v=0$ axis. These are points for which the best estimate of the velocity is near 0, but if a different value is used for the initial velocity guess it is possible to get a very large value for the final velocity. These are the points that are labeled Population A in the previous figure. It is difficult to see in Figure 5, but there is also a set of points clustered on the diagonals, where the velocity error is equal to the velocity itself. These are the points from Population B. They are the points where the best guess for the initial velocity results in a high velocity value, but one of the other guesses results in a near 0 velocity.

Bug fixes

In the process of implementing the corrected weighting scheme, the new definition of ground scatter, and the new algorithm for estimating the velocity error a number of bugs were found that exist in the current version of FITACF.

1. In moving from RADOPS 2000 to RST a bug was introduced into the calculation of width_s_err, the error estimate on width when using the gaussian decay model. The error resulted in the value of width_s_err to be a constant value. Clearly no one has been looking at width_s_err or this would have been immediately spotted.

2. In moving from RADOPS 2000 to RST a bug was introduced into the calculation of the elevation angle. This bug only affects two radars, Kapuskasing and Goose Bay. Because of the lateral offset of the interferometer at Goose Bay a special subroutine has to be used to calculate the elevation angle from Goose Bay. Unfortunately, in RST, the test to see if the special routine had to be called was to compare the station ID with 3. Of course, the station ID of Goose Bay is 1. As a result of this error, all Kapuskasing (station ID=3) elevation angle data were calculated with the Goose Bay routine and all the Goose Bay data were calculated with the standard routine. If we are going to make use of elevation angle data from these two stations all the FIT files will have to be reprocessed from the time RST was implemented at those sites. Under certain very special conditions it is also possible for the elevation angle calculation in the Goose Bay routine to go into an infinite loop if the data aren't really from Goose Bay. This may explain some mysterious situations where the radar at Kap seemed to get hung up.
3. A bug was introduced in the transition from QNX version 2 to QNX version 4 (that's a long time ago!). Mathematically, the variance of a quantity is positive definite. However, it is possible, when subtracting two large floating point quantities for numerical round-off error to result in the variance being a small negative value. The code in both the very old RADOPS for QNX 2 and version for QNX 4 had a test for this condition, but it was improperly implemented in the QNX 4 version. As a result, the QNX 4 software would, at times, attempt to take the square root of a negative number. Under QNX 2 this would have lead to a floating point exception and the program would have crashed. Under QNX 4 this lead to the SQRT routine quietly returning NaN (Not a Number). Subsequently, when the NaN was used in a calculation the end result ended up being set to 0.
4. When the estimate of an error parameter cannot be done because there are too few points in the curve, the code sets the error estimate to the specially identified quantity HUGE_VAL. We thought that any mathematical operation on HUGE_VAL would return HUGE_VAL, but apparently this is not always true. In some cases you end up with what should be HUGE_VAL actually being 0. The new version of FITACF tests for HUGE_VAL before it performs any calculation with a quantity that potentially can be HUGE_VAL.
5. When FITACF writes its data to a file it converts the double precision floating point values to 16-bit integers. We thought we had checked all the cases where there was a potential for an integer overflow to occur, but it seems that as the operating parameters have evolved we have new possibilities. The new version of FITACF tests all values before they are converted to integers. If the result would be an integer overflow, the result is set to the maximum positive (32767) or minimum negative (-32768) integer. Remember that the interpretation of this value depends on the parameter. For the velocity (for example) the maximum/minimum range becomes –3276.8 to +3276.6.

Conclusion and Recommendation

From the comparisons of the old results with the newer results it is clear that the new version of FITACF is not producing any major changes in the primary parameters. In addition, it fixes several bug and in particular corrects a major bug in the calculation for elevation angle and Goose Bay and Kapuskasing.

I believe that the new method for weighting the fits is correct. The change does not produce any significant changes in most of the primary parameters, but it does lead to some noticeable differences in the velocities (the Population A and Population B points in Figure 4). I also believe that the new method for estimating the error in the velocity is well founded mathematically and should be the method we use. Thus, I strongly recommend that these new aspects of FITACF should be implemented as soon as possible.

There remains the issue of what to do about the method of setting the ground scatter flag. First, I point out once again that this is only an issue of setting a flag. It does not change the value of any of the parameters. The user is free to perform his own determination of which points are and are not ground scatter. The original method of flagging ground scatter was at least based on a statistical analysis of the characteristics of the data. The method currently in use seems to have been implemented *ad hoc*. The new method is based on a statistical analysis of data, but some detailed analysis remains to be done and it is likely that a refinement of the algorithm will be made in the future. Finally, if we go ahead and use the new definition, should we use it with the basic ground-scatter function, *g*, or should the error estimate on *g* be used to extend the set of points identified as ground scatter? Figure 2c may look prettier than 2b, perhaps, but the justification for including the error correction is weak. I therefore recommend that we implement the new definition of the ground scatter flag without any error extension.

Recommendation:

Implement the full set of revisions to FITACF as soon as possible.