FITACF

A SuperDARN Tutorial

Kile Baker

Introduction

- **Multi-pulse Technique and ACFs**
  - Why ACFs?
  - Bad and missing lags
- **Basic Theory of FITACF**
- **Basic Fitting Technique**
  - Getting the Power and Width
  - Getting the Velocity
- **Dealing with Noise**
- **Error Estimation**
- **Future Directions**

Consider a Double Pulse

- By comparing the relative phase of the signal received at time t_0 (signal received from first pulse) with phase received at time $t_0 + \tau$ (signal from second pulse) we can determine by much much the irregularities have moved in time τ.

How do we integrate over many pulse sequences?

- Let S_{11} be the complex signal received from pulse sequence #1, pulse 1. Let S_{12} be the signal received from pulse sequence #1, pulse 2.
  - $$S_{11} = A_1 \exp(i\varphi_1) \qquad S_{12} = A_1 \exp(i\varphi_1 + i\omega\tau)$$

- Let S_{21} be the complex signal received from pulse sequence #2, pulse 1. Let S_{22} be the signal received from pulse sequence #2, pulse 2.
  - $$S_{21} = A_2 \exp(i\varphi_2) \qquad S_{22} = A_2 \exp(i\varphi_2 + i\omega\tau)$$

We can't simply sum the two because of the random phase.

Double Pulse ACF

$$R_{1,0} = S_{11}S_{11}^* = (A_1 e^{i\varphi_1})(A_1 e^{-i\varphi_1}) = A_1^2$$

$$R_{1,1} = S_{12}S_{11}^* = (A_1 e^{i(\varphi_1 + \omega\tau)})(A_1 e^{-i\varphi_1}) = A_1^2 e^{i\omega\tau}$$

Since the ACF is independent of the random phase we can sum ACFs from different pulse pairs to get a sensible result.

Range Aliasing

- Suppose our two pulses are separated by time τ

- Suppose we also have two regions of backscatter and they are separated by a distance, $\mathbf{d=c\tau/2}$

- Then, we will get a signal back from range #2 scattering pulse #1 at the same time as we get a signal from range #1 scattering pulse #2.

Range Aliasing and ACFs

Signal from pulse 1

$$S_{11} = A_1 e^{i\varphi_1}$$

$$S_{12} = A_1 e^{i(\varphi_1 + \omega\tau)} + B_1 e^{i\vartheta_1}$$

$$S_{21} = A_2 e^{i\varphi_2}$$

$$S_{22} = A_2 e^{i(\varphi_2 + \omega\tau)} + B_2 e^{i\vartheta_2}$$
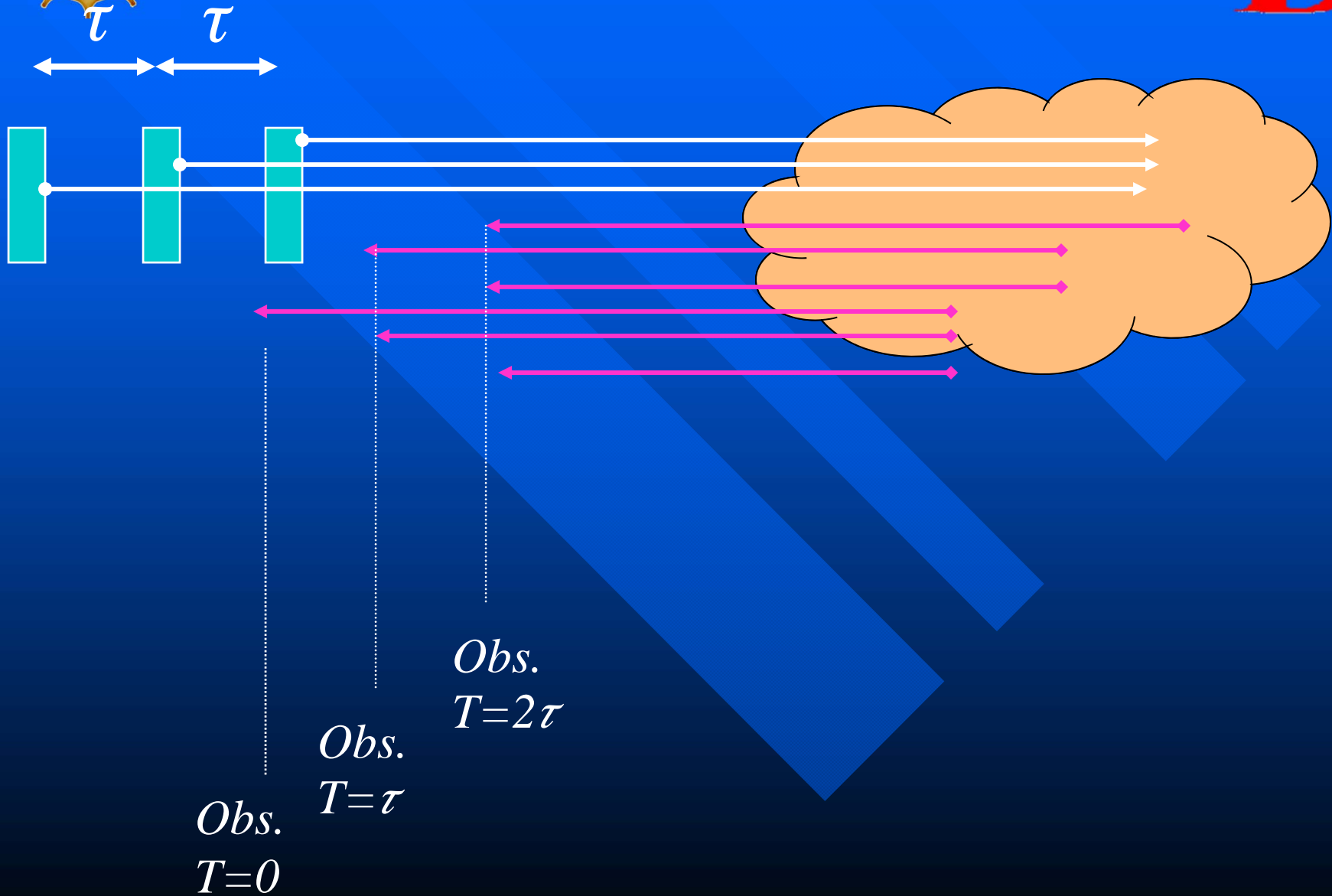

$$R_{1,1} = S_{12} S_{11}^* = A_1^2 e^{i\omega\tau} + A_1 B_1 e^{i(\vartheta_1 - \varphi_1)}$$

$$R_{2,1} = S_{22} S_{21}^* = A_2^2 e^{i\omega\tau} + A_2 B_2 e^{i(\vartheta_2 - \varphi_2)}$$

Random phases in second terms will tend to cancel

What about a triple pulse?

τ τ

Obs.
T=0

Obs.
T=τ

Obs.
T=2τ

$$S_1 = Ae^{i\varphi}$$

$$S_2 = Ae^{i(\varphi+\omega_1\tau)} + Be^{i\vartheta}$$

$$S_3 = Ae^{i(\varphi+2\omega_1\tau)} + Be^{i(\vartheta+\omega_2\tau)} + Ce^{i\xi}$$

Now have 2 ways to calculate R_1:

$$R_1 = S_2 S_1^*$$

$$R_1 = S_3 S_2^*$$

This represents the general case. Any time we make a measurement we can be receiving signals from all three pulses if there is scatter coming from all possible ranges.
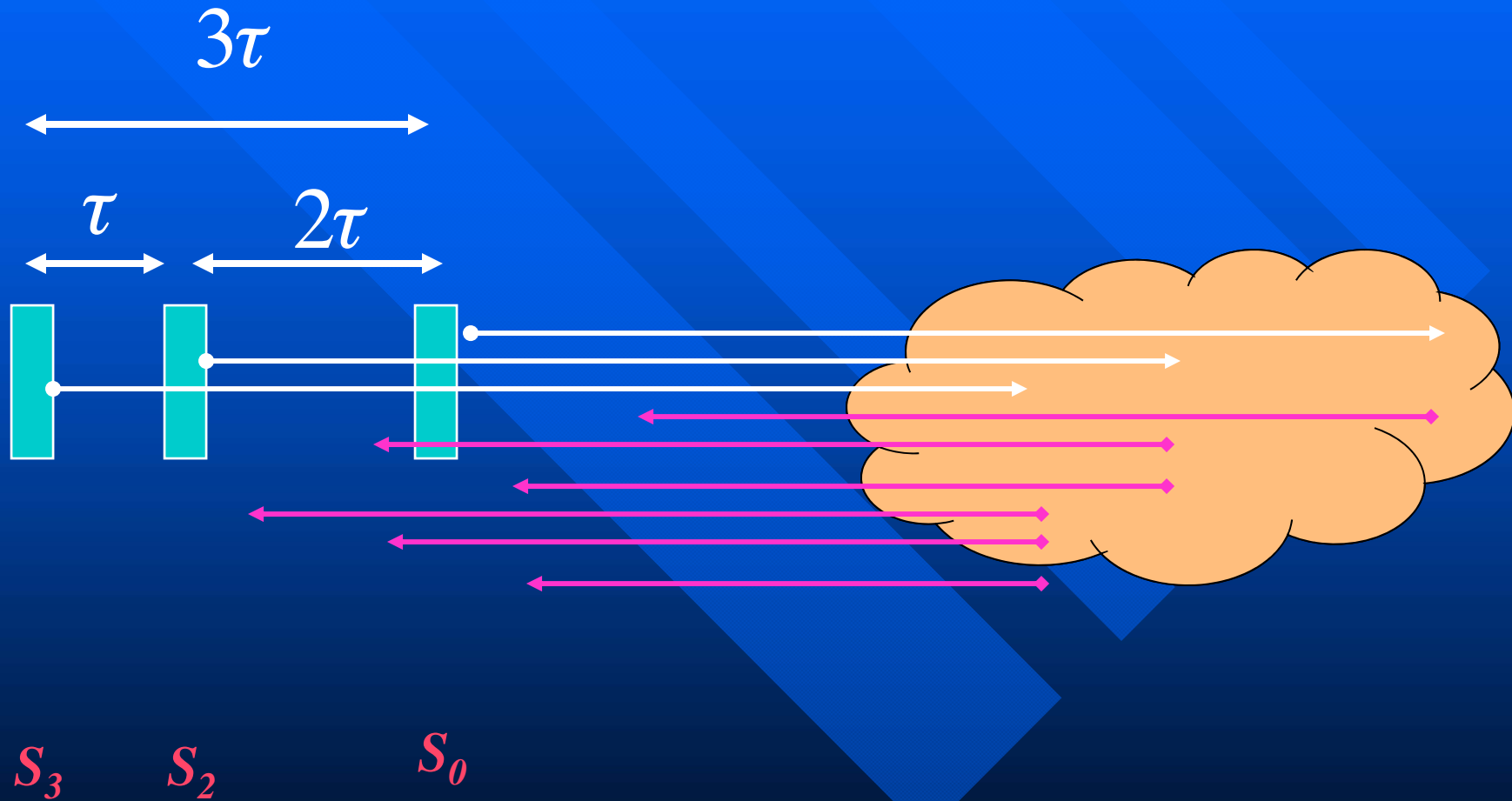
R_1 for triple pulse

$$R_1 = S_3 S_2^*$$

No random phase

$$R_1 = A^2 e^{i\omega_1 \tau} + B^2 e^{i\omega_2 \tau} +$$

$$AB[e^{i(\varphi - \vartheta + 2\omega_1 \tau)} + e^{i(\vartheta - \varphi + \omega_2 \tau - \omega_1 \tau)}]$$

$$ACe^{i(\xi - \varphi - \omega_1 \tau)} + BCe^{i(\xi - \vartheta)}$$

Three Pulse Pattern

ACF from Multi-pulse Pattern

…. Indicates terms with random phase.

$$R_0 = S_3 S_3^* = A^2 + \ldots\ldots$$

$$R_1 = S_3 S_2^* = A^2 e^{i\omega_1 \tau} + \ldots\ldots$$

$$R_2 = S_2 S_0^* = A^2 e^{i2\omega_1 \tau} + \ldots\ldots$$

$$R_3 = S_3 S_0^* = A^2 e^{i3\omega_1 \tau} + \ldots\ldots$$

Repeated Pulse vs Multi-Pulse Pattern

- Sequence of repeated pulses gives you all the different lags of an ACF - BUT you get range aliasing for every lag and it does NOT go away.

- Multi-pulse pattern solves the range aliasing problem - BUT
  - You cannot have every lag UNLESS
  - You have redundant lag pairs, which causes range aliasing again.
  - It only solves the problem to the extent that terms with random phase go away as we average a large number together.

- We try to minimize redundancy, but this means we always have missing lags.

Bad Lags (1)
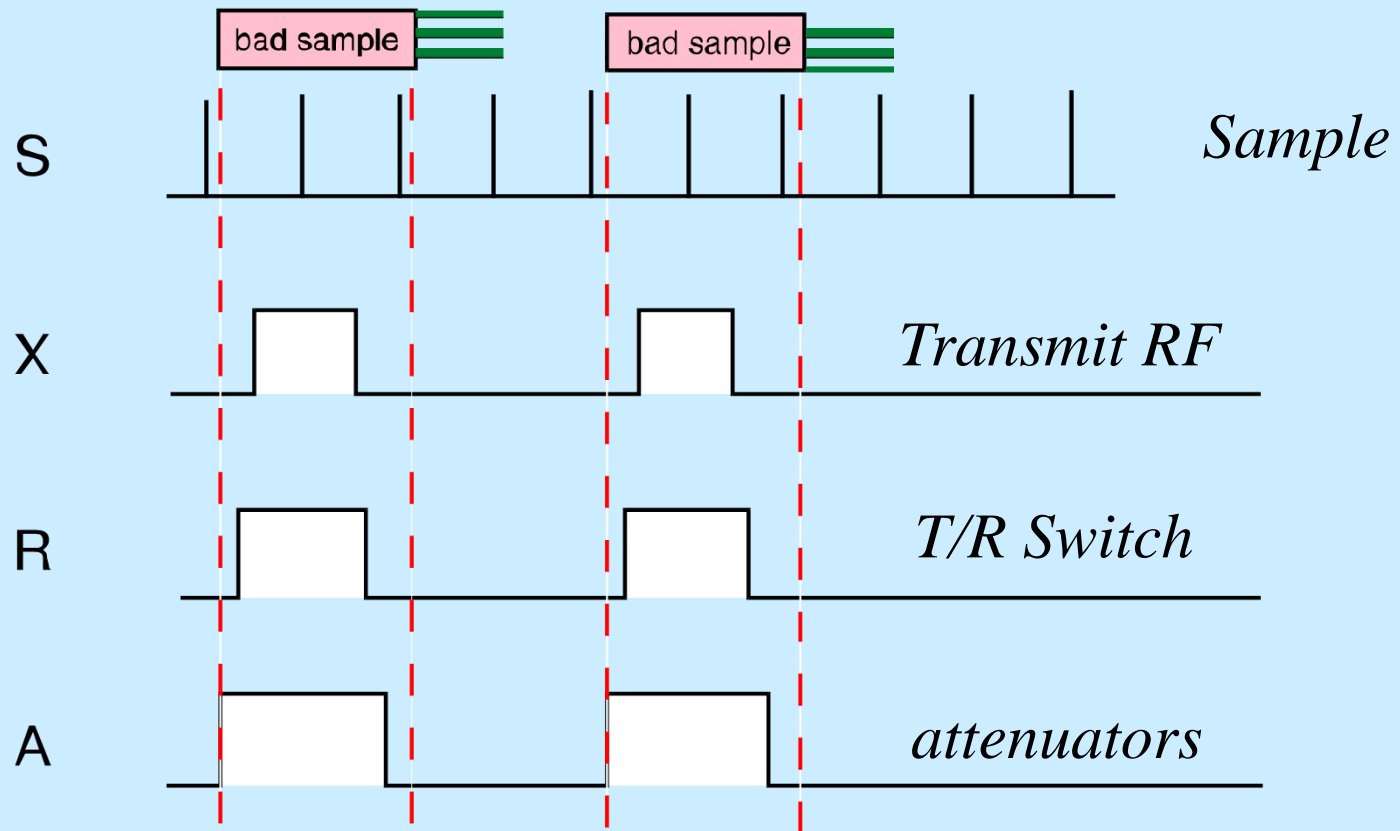
Section of a timing sequence

Bad lags (2)

- **General form of a lag in an ACF from a single pulse sequence is:**
  - $R = A^2 \exp(i\omega\tau) + AB \exp(\text{random phase})$
- **When we average ACFs from N pulse sequences**
  - $R = A^2 \exp(i\omega\tau) + AB/\sqrt{N}$
  - If the power from range B is significantly greater than the power from range A, then we get contamination of the ACF from unwanted power at range B, even though the contamination is decreasing as $1/\sqrt{N}$.

Bad Lags (3)

- An ACF characterized by a single frequency should have its maximum value at lag-0 and each successive lag should have $pwr(n) \leq pwr(n-1)$.
  - Allow for the possibility of a small increase due to the effect of noise.
  - If condition is not met then identify the lag as a bad lag.
    - » The presumption is that the lag has been affected by a burst of noise.
    - » But it might really be due to a failure of the assumption that the ACF is characterized by a single frequency.

Finally – we come to FITACF

- Goose Bay Radar was built in Oct. 1983
- In 1984 Christian Hanuise developed the first code to attempt to determine the velocity by fitting all the lags of an ACF.
- 1985 – after Hanuise returned to France, improved versions of Hanuise's code were developed.
- 1985 or 86? The first version of FITACF was produced.
  - Improved flexibility
  - Improved error estimation
 - Introduction of estimation of spectral width

Basic Theory of FITACF
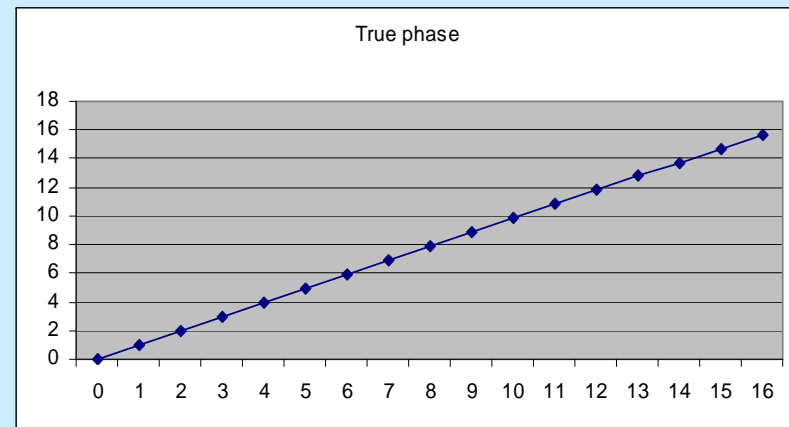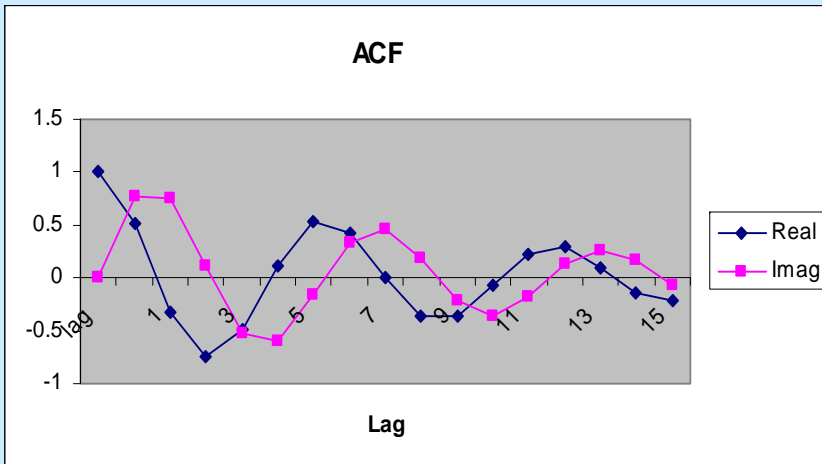
- The ACF can be characterized by a single frequency

- The ACF will decorrelate as the lag increases. We will model the decorrelation two different ways:
  - Power $\sim \exp(-\lambda t)$ Log(pwr) decreases linearly
  - Power $\sim \exp(-\sigma^2 t^2)$ Log(pwr) decreases quadratically

- Convolution Theorem: Fourier transform of an ACF is the power spectrum.

Example of a perfect ACF
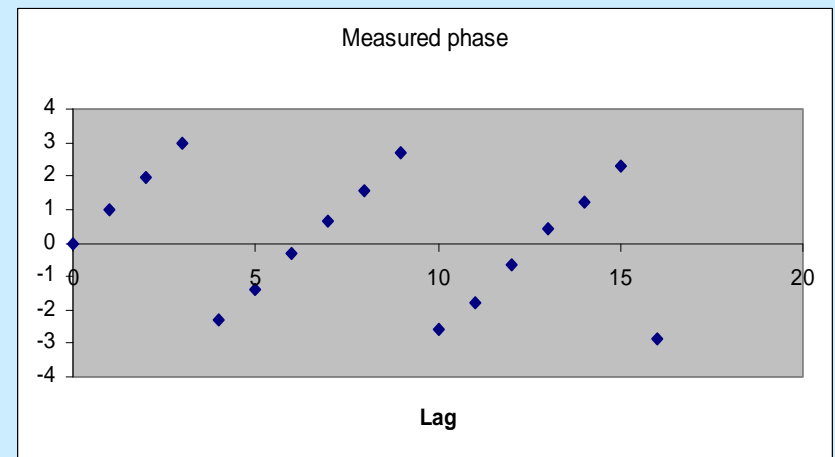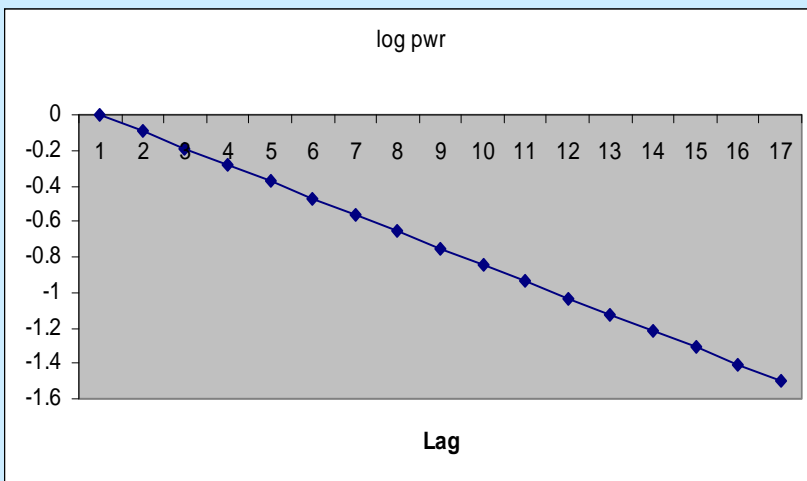
Why bother with the ACF? Why not just work with the power spectrum?

- Getting the velocity (frequency) is difficult in the power spectrum. Unless the true frequency is an exact value of the discrete Fourier transform, you have leakage from the central peak to other points in the FFT.

- The leakage problem also makes it difficult to accurately determine the spectral width from the spectrum.

- We always have missing lags and often have bad lags. This complicates the calculation of the power spectrum from the ACF and makes interpretation of the power spectrum more difficult.

ACF Model

$$R = Ae^{i\omega t}e^{-\lambda t}$$

$$R = Ae^{i\omega t}e^{-\sigma^2 t^2}$$

Lambda Model

Sigma Model

$$Power = |R| = Ae^{-\lambda t}$$

$$|R| = Ae^{-\sigma^2 t^2}$$

$$\tan(\omega t) = \frac{\text{Im}(R)}{\text{Re}(R)}$$

NOTE: The Power and the Phase are completely independent

Fitting Technique

- Conceptually things are now straightforward.
  - Take Log(power) and do a Least Squares Fit
    - » Log(pwr) $= A - \lambda t$ *for Lambda Model*
    - » Log(pwr) $= A - \sigma^2 t^2$ *for the Sigma Model*
  - Calculate *true* phase at each lag and do a Least Squares FIT with condition that phase at lag-0 must be 0.
    - » $\varphi = \omega t$

- Bad or missing lags in the ACF are simply ignored in the fitting process.

- No leakage problems like there are when dealing with the power spectrum.

The lambda power spectrum

$$\overline{R}(\omega) = \int\limits_{-\infty}^{\infty} e^{i\omega_0 t} e^{-|\lambda|t} e^{-i\omega t} dt$$

The real part is an even function while the imaginary part is an odd function. The imaginary part therefore integrates to 0 and for the real part we can write:

$$\overline{R}(\omega) = 2\operatorname{Re}\int\limits_{0}^{\infty} e^{-\lambda + i(\omega_0 - \omega)t} dt =$$

$$\operatorname{Re}\left[-2\frac{1}{-\lambda + i(\omega_0 - \omega)}\right]$$

We now multiply the numerator and denominator by the complex conjugate of the denominator and then take the real part of the result.'

This gives us the power spectrum as a function of ω and λ.

$$\bar{R}(\omega) = \frac{2\lambda}{\lambda^2 + (\omega_0 - \omega)^2}$$

This is a Lorentzian power spectrum with a maximum at ω_0. To determine the full width at half-max, simply solve for the value of ω that gives $R(\omega) = 1/\lambda$.

$$\omega_{1/2} = \omega_0 \pm \lambda$$

We do the same thing to get the spectrum and spectral width for the Gaussian decay model.

$$R(\omega) = \int_{-\infty}^{\infty} e^{[-\sigma^2 t^2 + i(\omega_0 - \omega)t]} dt$$

To integrate this we look for a quadratic such that

$$-[\sigma t - q]^2 = -\sigma^2 t^2 + i(\omega_0 - \omega)t + k$$

This gives us

$$q = \frac{i(\omega_0 - \omega)}{2\sigma}, q^2 = -\frac{(\omega_0 - \omega)^2}{4\sigma^2}$$

We can then write the power spectrum as:

$$R(\omega) = \int_{-\infty}^{\infty} e^{-(\sigma t - q)^2} e^{q^2} \, dt$$

Then perform the variable substitution: $x = \sigma t - q$, $dx = \sigma dt$

$$R(\omega) = \frac{e^{q^2}}{\sigma} \int_{-\infty}^{\infty} e^{-x^2} \, dx = \frac{e^{q^2}}{\sigma} \sqrt{\pi}$$

Substituting in the value of q^2 we then have

$$R(\omega) = \frac{\sqrt{\pi}}{\sigma} e^{-(\omega_0 - \omega)^2 / 4\sigma^2}$$

Getting Physical Parameters from λ, σ

Power Spectrum

$$\overline{R}(\omega) = \frac{2\lambda}{\lambda^2 + (\omega_0 - \omega)^2}$$

$$\overline{R}(\omega) = \frac{\sqrt{\pi}}{\sigma} e^{-(\omega_0 - \omega)^2 / 4\sigma^2}$$

Full width at ½ max

$$W_\lambda = 2\lambda$$

$$W_\sigma = 4\sigma\sqrt{\log(2)}$$

The relation between the Doppler shift and the velocity is usually given by:

$$\frac{\Delta f}{f} = \frac{v}{c}$$

However, for a Doppler shift produced by a reflection from a moving target, the effective velocity is twice the true velocity so we have: *Now, ω is the angular frequency*

$$\frac{\Delta f}{f} = \frac{2v}{c}$$

of the Doppler shift, so $\omega = 2\pi\Delta f$, and we then have (next page)

Velocity and width in terms of velocity

$$v = \frac{c}{4\pi f}\omega$$

Where v is velocity in m/s, c is speed of light, f is transmitter frequency in Hz, ω is radial Doppler shift frequency from the phase fit.

The same conversion factor is used to convert the spectral width from frequency to velocity.

$$W_\lambda = \frac{c}{4\pi f}(2\lambda) \qquad W_\sigma = \frac{c}{4\pi f}(4\sigma\sqrt{\log(2)})$$

Complications

- Should we use lags where the power in the ACF is less than the noise level? NO.

- What IS the noise level?

- Is the error associated with the measurement of each lag the same? If not, how do we weight the points when doing the fit?

- How do we determine the *true* phase from the measured phase?

Determining the True Phase

- Measured phase is between $+\pi$ and $-\pi$
- In the ACF example it was obvious where the jumps in phase occurred but
  - When we add noise it's not so obvious
  - When we don't have all the lags it's not so obvious
- We have to assume that the phase for lag-1 is between $-\pi$ and $+\pi$.
- Start with an initial guess for ω and do the fit one point at a time. Predict phase and then adjust.

Guess, Predict, Adjust

- **How do we make the initial guess?**
  - The basic assumption is that between neighboring lags (e.g. lag-4 to lag-5) the phase change must lie between $-\pi$ and $+\pi$.
  - Find all good 1-lag pairs and calculate the average phase difference. Then $\omega_0 = <\Delta\varphi>/\tau$.

- **Go lag by lag:**
  - Predict the true phase at lag-n from $\omega_{n-1}(n\tau)$
  - Compare the predicted phase with $\varphi_{meas} + 2k\pi$
    - » Adjust k so that the result lies with $+/- \pi$ of the predicted value. This is the *true* value of the phase.
  - Use the new true value of φ to calculate ω_n (least squares fit).

Check and Iterate

- If you have a reasonably good guess for the initial value of ω, you are not terribly bothered by missing or bad lags. This is because you are allowed to jump the phase up (down) by more than one multiple of 2π.

- Check the final result for ω by redoing the fit using that value of ω as the initial guess.
  - Iterate until you get a new value of ω that is the same as the previous value. Usually this only takes one pass.

Dealing with Noise

- **Types of noise**
  - Random cosmic noise
    - » We have never really investigated the statistical properties of this cosmic noise. **This would be really good to do!**
  - Bursts from other transmitters
  - CW noise from other transmitters
- **Is there a noise ACF that can be removed?**
  - If we find CW noise from other transmitter it may be possible to (partially) remove the signal
- **Noise and decorrelation**
- **Weighting the fits**

Cosmic Noise

- Things that would be nice to know about the cosmic noise (but we don't)
  - Does the power truly behave like a random variable? If so, what is it's distribution?
  - Does the measured phase of the noise behave like a random variable? What is it's distribution?
  - Is there a clear diurnal variation in the noise?
  - Can the radars be used as oblique riometers by measuring the noise?

Determining the noise characteristics

- During Clear Frequency Search we determine a noise level characteristic of the clearest frequency during the brief period of the search. That value is saved in the NOISE parameter in the parameter block.
- In FITACF
  - Find the average lag-0 power for the 10 lowest power ranges. We call these 10 lowest power ranges the "noise range gates." This value is saved in the parameter "skynoise" in the FIT files.
  - Find the average noise from all lags (not just lag-0) for the noise range gates. This value is called the "noise_pwr" and is used in the fitting process.
    - » Compare this value **and its variance** to the skynoise to see if there appears to be a coherent signal present in the noise range gates. If so, we assume that a CW transmitter has contaminated all range gates.
    - » If there is a coherent noise signal, calculate the average ACF for all noise range gates and then subtract that "noise ACF" from each ACF.
    - » If you subtract a noise ACF, recalculate all the noise parameters.

Noise and Decorrelation

- If you could pick out the true ACF from a single pulse sequence, there would hardly be any decorrelation. The decorrelation comes about because parameters such as frequency and spectral width have some random character and change from pulse sequence to pulse sequence.

- The ideal ACF is only achieved in the limit of a large number of pulse sequences in the average.

- When looking at the Power fit, there is an artificial noise level determined by the lag-0 power, the random characteristics of frequency and spectral width, and the number of pulse sequences. This is often higher than the true noise level.
  - We may not use the same number of lags in the phase fit and the power fit.

Weighting the Phase Fit

- You can show that the error in the phase measurement at any given lag is proportional to Noise/Signal.
  – This implies that the phase fit should be weighted by the power at each lag (that is, the signal strength at that lag).

- The error in the power at any given lag is constant (proportional to the Noise level),
  – BUT when we take the log of the power you again find that the error is proportional to N/S (assuming S>>N).
  – So again, the power fit should be weighted by the power at each lag.

NOTE: the weighting is by the actual power in the lag, NOT the log of the power.

Error Estimates

- The method for estimating the error in fitted parameters from a least-squares fit is well established and can be found in many books. The error estimate for a 2-parameter linear fit (such as the power fit) is obtained from an overall estimate of the error in the fit and the covariance matrix.

- Very straightforward for the power fit

- We have a complication, however, with the phase fit.

Error in the phase fit (velocity error)

- If we had no 2π ambiguities the process would be just as straightforward as in the power fit.
- Suppose when we calculate our initial guess for ω we find that the variance in the 1-lag phase differences is large.
  - It is then, very likely that we will have a small value of ω.
  - If the predicted value at each lag for the phase always lies within $+/- \pi$, we will get a small velocity with a somewhat large error (comparable to the velocity estimate itself) but the true velocity may be a much larger value.

$$\omega_0 = \frac{\sum_{k=1}^{M}\left(\frac{\Delta\varphi_k}{\tau}\right)W_k}{\sum_{k=1}^{M}W_k}$$

$$\left\langle\omega_0^2\right\rangle = \frac{\sum_{k=1}^{M}\left(\frac{\Delta\varphi_k}{\tau}\right)^2 W_k}{\sum_{k=1}^{M}W_k}$$

$$(\Delta\omega_0)^2 = \left\langle\omega_0^2\right\rangle - (\omega_0)^2$$

$$\omega_n = \frac{\sum k \varphi_k W_k}{\tau \sum W_k k^2}$$

Value of ω using all good lags. Weighted by Power at each lag

$$(\Delta \omega_n)^2 = \frac{\sum W_k (\varphi_k - \omega_n k \tau)^2}{\sum W_k}$$

Error estimate for ω based solely on the phase fit.

Final estimate of frequency error

- We now have two different errors related to the frequency.
  - Error estimate on ω_0, the initial guess
  - Error estimate on ω_n, from the phase fit
- Combine the two using the geometric mean

$$\Delta\omega = \sqrt{(\Delta\omega_0)(\Delta\omega_n)}$$

Is there a better way?

- If we have a good initial guess then that implies we are putting the 2π's in the right place. And that means we should be using the error estimate from the phase fit and not the geometric mean of the error in the initial guess and the phase fit error.

- Suppose we re-did the fitting process but used as our initial guess $\omega_0 + \Delta\omega_0$? (and similarly for $\omega_0 - \Delta\omega_0$).
  - If, after going through this process we ended up with the same final value of ω then our initial guess was "right" and we should use the error estimate from the phase fit.
  - If they aren't the same, then let $\Delta\omega = \omega_+ - \omega_-$.

- The downside of this is that we triple (at least) the amount of computer time needed for the phase fit.

Recommendation: Some one should try this using real data from DAT files.

Future?

- **Dealing with mixed frequencies (particularly ionospheric scatter + ground scatter)**
  - MUSIC? (Andre and Villain)
  - Subtract mean value of Re(ACF)? (only for ground scatter, causes errors when ionospheric velocity is low, could mess up spectrum)
  - Fourier transform?
    - » Identify ranges that have the problem *Can we do this reliably?*
    - » Remove unwanted peak(s) *Unwanted by whom?*
    - » Transform back

More for the future

- Improved recognition of ground scatter
  - FITACF has changed a lot since our definition of ground scatter was made.
- FITACF treats each range completely independently? Could we improve things by including information from neighboring ranges? What about neighboring beams?
  - Would we have to analyze data differently depending on which direction the radar was oriented?
  - If we try to use information from neighboring beams, how do we handle non-standard scan modes?

Dealing with raw data?

- Currently the only way we deal with raw data is by calculating ACFs. Shouldn't we investigate alternate approaches?

- We should have some truly RAW data – in other words, the actual samples from the A-D converter, totally unprocessed.

Conclusion

FITACF

It's been a good approach for many years, but that doesn't mean improvements can't be made. But improvements should be done very carefully and tested thoroughly before they are implemented.