External noise effects on lag 0 ACF/XCF power and phase

- **ACF lag 0 parameters**

An important point to make here is that even in the absence of external noise the received backscatter signal will vary due to its intrinsically statistical nature resulting from a collective scatter process involving a large number of plasma irregularities filling in the effective scatter volume. At lag 0 the signal is characterised by the mean power, P_s and the standard deviation, σ_s. The standard deviation is a function of the power and the number of averages, N:

$$\sigma_s = \frac{P_s}{\sqrt{N}}$$

Adding noise increases both mean power (offset) and statistical variance. For the mean power, it is a simple sum of the signal power and the noise power,

$$P_{s+n} = P_s + P_n$$

This leads to a delta-function "spike" in ACF power at lag 0. This effect is mitigated in both FITACF2 and FITACF3 by subtracting the mean noise power from lag 0.

The standard deviation is also increasing proportionally

$$\sigma_{s+n} = \frac{P_s + P_n}{\sqrt{N}}$$

It is easy to see that the noise effect on the statistical variability becomes significant when the noise power becomes comparable with that of the signal, i.e., when $SNR \leq 1$, and normally these data are removed from analysis. For example, $SNR > 1$ criterion is used for FITACF3 data preselection.

There is an important difference in effects of noise on non-zero lags as compared to those on lag 0: the power offset occurs only at lag 0 while the statistical fluctuations increase across all ACF lags by a similar amount, i.e., $\frac{P_n}{\sqrt{N}}$ for lag 0 and $\frac{P_n}{\sqrt{2N}}$ for non-zero-lags.

The lag 0 ACF phase mean and standard deviation values are always zero, $\varphi = 0, \sigma_\varphi = 0$, owing to the fact that the signal is correlated with itself with zero time offset so that even the noise component is fully correlated.

- **XCF lag 0 parameters**

For XCFs there are some additional factors that need to be taken into account while analysing the noise effect on the lag 0 power and phase. The most important one is that the interferometer antenna beam is significantly wider than that of the main antenna array. This means that the interferometer channel receives comparatively larger amount of noise so that the effective signal-to-noise ratio is consequently lower.

It would be relatively easy to estimate the noise effects on XCF power and phase if the noise measurements from the main and interferometer arrays were uncorrelated. However, in our case the noise is partially correlated as the main array beam is embedded into the interferometer array one. The solution in this situation requires knowledge of the cross-correlation coefficient at lag 0,

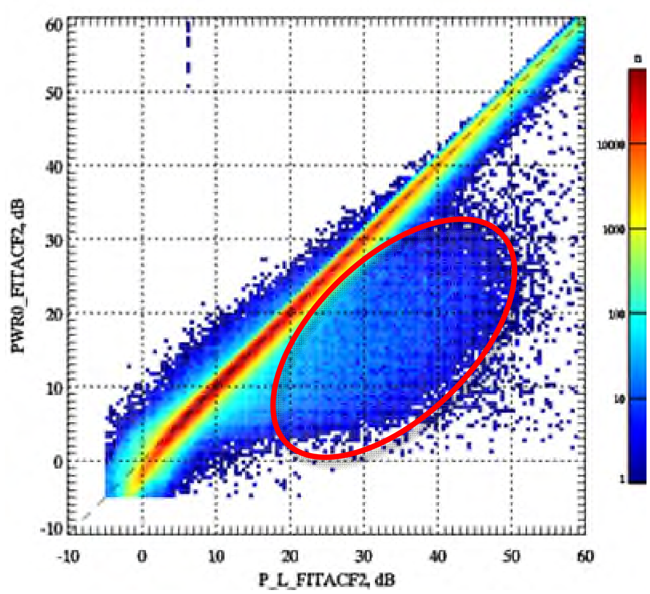
$$r_{AI}(0) = \frac{R_{AI}(0)}{\sqrt{R_{AA}(0)R_{II}(0)}}$$

where $R_{AA}(\tau), R_{II}(\tau)$ are auto-covariance functions of signals from the main (A) and interferometer (I) antenna arrays, respectively, and $R_{AI}(\tau)$ is a cross-covariance function between those two signals. While $R_{AA}(\tau)$ and $R_{AI}(\tau)$ are available from the SuperDARN data – these are what we conventionally call ACF and XCF –, there is no information on $R_{II}(\tau)$ currently stored in standard SuperDARN datasets. In principle, $R_{II}(\tau)$ can be recovered from I&Q data, but there is no way to obtain it from RAWACF data.

As a result, right now it is hard to get accurate estimates of the external noise effects on lag 0 XCF power and phase.

- **Comparison with fitted values**

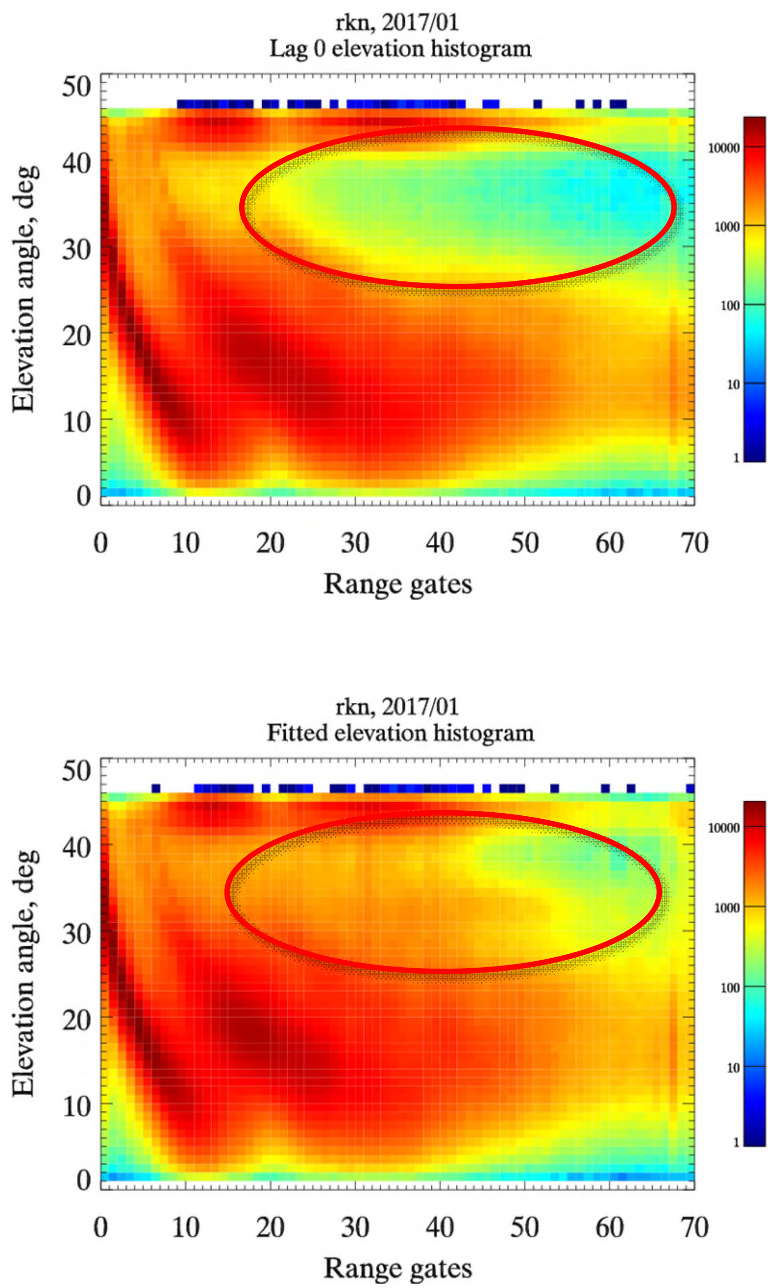
In principle, utilising all available lags should improve the statistical validity in estimating lag 0 ACF/XCF parameters as compared to the directly measured values as a larger statistical ensemble is used. However, the reliability of the fitted estimates depends on a correct/optimal implementation of the fitting procedure, which is generally not the case with FITACF1-2. For example, the non-optimal weighting coefficients in the least-square fitting sums implemented in FITACF2.5 most probably cause the observed spread of the fitted power values, *p_l*, as compared to the directly measured lag 0 power, *pwr0*:



This effect has been effectively mitigated in FTACF3through optimal weighting of fitted data.

Furthermore, as it has been shown experimentally for both FITACF2.5 and FITACF3.0, elevation angle values obtained from the directly measured lag 0 XCF phase (top panel below) are actually more consistent, i.e., show less spread, than those obtained through fitting a linear function to the non-zero lag XCF phase values (bottom panel):

FITACF3.0



The most probable causes of this difference are (i) imperfect phase “unwrapping” and/or (ii) CRI. Importantly, neither of these factors affects the directly measured lag 0 cross-phase.

- **Conclusions**

(1) External noise effects on lag 0 ACF power are well understood and can be easily quantified. They do not seem to be too serious for the bulk of utilised data (SNR >1).

(2) External noise effects on lag 0 XCF power and phase are qualitatively well understood too. However, right now they cannot be accurately quantified as the important information on the interferometer ACF is currently unavailable from the standard SuperDARN data products.

(3) Using fitted values of ACF power or elevation instead of the directly observed values is not always advantageous as the reliability of the fitted estimates depends on a number of factors such as correct implementation of the fitting process, effectiveness of CRI mitigation, accurate unwrapping of XCF phase etc.